EE 103 Lecture 7
April 17, 2017 (M)

Correction Tutor email kjohns07@ucsc.edu

Reading Section 4.1 lightly
4.2 & 4.3

HW(WK3)
1) prob 4.1
2) prob 4.2 (i)
3) prob 4.3 (iv)
4) prob 4.6 for P4.6(b) only
5) prob 4.12 (a)
6) prob 4.12 (f)

Quiz 2 at 2:10 pm today (15 min.)
For this quarter there will be 8 quizzes.
May 8 is the Midterm day
You will be excused 1 out of 8 quizzes.

EE 103 grading

Quiz 20% $\left(\sum_{k=1}^{7} x_k \times \frac{100}{70} \times 0.2\right)$, $x_k \le 10$
Midterm 30% $(y \times 0.3)$
Final 50% $(z \times 0.5)$
100%

Let us consider when $x(t) = e^{st}$

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) e^{s(t-\tau)}\, d\tau = e^{st} \int_{-\infty}^{\infty} h(\tau) e^{-s\tau}\, d\tau$$

$$= e^{st} H(s)$$

where $H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau}\, d\tau$

Laplace Transform

So, $y(t) = x(t) \times$ Laplace transform of $h(t)$
for $x(t) = e^{st}$

$e^{st} \rightarrow \boxed{\int} \rightarrow y(t) = \int_{-\infty}^{t} e^{s\tau}\, d\tau = \frac{1}{s} e^{s\tau}\Big|_{-\infty}^{t} = \boxed{\frac{1}{s}} e^{st}$

$\frac{1}{s} = H(s)$

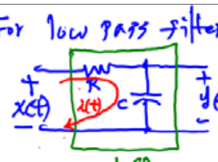
$e^{st} \rightarrow \boxed{\frac{d}{dt}} \rightarrow y(t) = \frac{d}{dt} e^{st} = \boxed{s} e^{st}$

$s = H(s)$

$e^{st} \rightarrow \boxed{K} \rightarrow y(t) = \boxed{K} e^{st}$

$K = H(s)$

For low pass filter

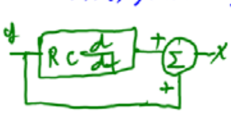


$$y(t) = \frac{1}{c}\int_{-\infty}^{t} i(\tau)\, d\tau \quad (1)$$

$$x(t) = R i + \frac{1}{c}\int_{-\infty}^{t} i(\tau)\, d\tau \quad (2)$$

From (1) $i(t) = c\frac{dy(t)}{dt}$ (3)

(3)→(2) yields $x(t) = RC\frac{dy(t)}{dt} + y(t)$

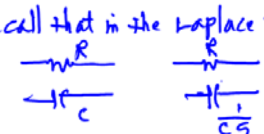


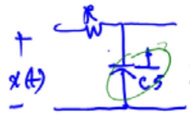
$= RC(s\, y(t)) + y(t)$

$= (RCs + 1)\, y(t)$

$y(t) = \underbrace{\frac{1}{RCs+1}}_{H(s)} x(t)$ for $x(t) = e^{st}$

Recall that in the Laplace Transformed domain





$$y(t) = \frac{\frac{1}{cs}}{\frac{1}{cs} + R} x(t)$$

$$= \underbrace{\boxed{\frac{1}{RCs+1}}}_{H(s)} x(t)$$

Revisit of BIBO stability

$x(t) \to \boxed{h(t)} \to y(t) = x(t) * h(t)$

$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(t-\tau)\, \underline{h(\tau)}\, d\tau$

$|y(t)| = \left| \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau \right|$

$\uparrow$ bounded $\forall t$ $= \int_{-\infty}^{\infty} \underbrace{|x(t-\tau)|}_{\text{bounded } \forall t} |h(\tau)|\, d\tau$

$\le X \underbrace{\int_{-\infty}^{\infty} |h(\tau)|\, d\tau}_{\text{bounded}}$

$h(t)$ is BIBO stable if and only if $\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$

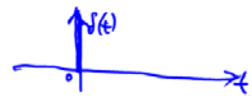
Integrator is NOT BIBO stable

$\delta(t) \to \boxed{\int} \to u(t)$

$h(t) = u(t)$

$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} 1\, dt = t \Big|_{-\infty}^{\infty}$ is not bounded.

$\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = \int_{-\infty}^{t} \delta(\tau)\, d\tau = 1 \quad \forall t \quad t \ge 0$

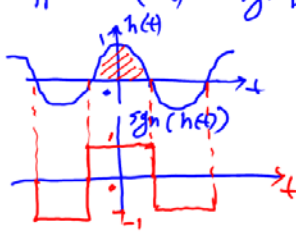


$\delta(t) \to \boxed{\int} \to y(t)$

$y(t) = \int_{-\infty}^{\infty} \delta(\tau)\, d\tau \quad \forall t$

e.g. $h(t) = \cos 2\pi t$

Suppose $x(-t) = \operatorname{sgn}[h(t)]$



$x(-t) = x(t)$

$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} \underline{\operatorname{sgn}(h(t-\tau))} \cdot \cos 2\pi\tau\, d\tau$, $\operatorname{sgn}(h(t-\tau)) = \operatorname{sgn}[\cos 2\pi(t-\tau)]$

$|y(t)| = \int_{-\infty}^{\infty} \underbrace{|\operatorname{sgn}(h(t-\tau))|}_{=1} |\cos 2\pi\tau|\, d\tau$

NOT BIBO stable